Bending Moment and Shear Force Diagram (B.M.D. & S.F.D.)

B.M. & S.F. are internal reactions in a structure.

* Shear force at a section means net vertical force either leftward or rightward of the section.
Direction of shear force at the section will be opposite of the net vertical force either leftward or rightward.

* Bending moment at a section means net moment of the forces at the section. Direction of B.M. at the section will be opposite of the net moment.

### Sign Conventions

- **Net Vertical Force**:
  - If upward, B.M. is +ve.
  - If downward, B.M. is -ve.

- **Shear Force**:
  - If rightward, S.F. is +ve.
  - If leftward, S.F. is -ve.

### Diagram Notes

- Whenever net vertical force is upward, B.M. at the shear force at the section is +ve. Section is making a clockwise couple making the structure sagging. B.M. is +ve when causing sagging in the structure.

- Whenever net vertical force is downward, B.M. at the shear force at the section is -ve. Section is making a counterclockwise couple making the structure hogging. B.M. is -ve when causing hogging in the structure.
Relationship b/w Intensity of load, S.F. & B.M.

\[ \frac{dW}{dx} = -w(x) \]

Slope of Shear = -(intensity of Distributed load)

Diagram

\[ \frac{dM}{dx} = V \]

Bending

Slope of Moment = S.F.

Diagram

\[ \Delta V = -\int w(x) \, dx \]

Change in Shear = (Area under distributed loading diagram)

\[ \Delta M = \int V(x) \, dx \]

Change in = Area under S.F.D.

Bending Moment
Note -

0. Max. B.M. will be at s.f. = 0, or where s.f. is min.
1. Contraflexure points are points where B.M. changes sign.
2. \( \frac{dM}{dx} = Vs.f. \), i.e., Rate of change of B.M. can be found by \( V \) of s.f.
3. Where B.M. is Max (either +ve or -ve), s.f. will be zero.
4. S.F. suddenly changes at a point load or reaction
5. B.M. suddenly changes at a externally applied moment or couple.
Finding Reactions at A & B.

Assume directions of R_A and R_B as shown in Fig.

For finding Reaction at A: \( +\sum M_B = 0 \)

\[ R_A \times 180 - 100 \times 100 \times (40 + 50) + 100 \times 60 = 0 \]
\[ R_A = 4666.667 \text{ kg} \]

By \( +\sum V = 0 \)
\[ R_A + R_B - 100 \times 100 - 1000 = 0 \]
\[ R_A + R_B = 11000 \]
\[ R_B = 6333.333 \text{ kg} \]

5-F-D:

(a) In AD (from A to just left of D)

\[ V_{AD} = R_A \quad (+ \text{ve}) \]

(b) In DE (from D to just left of E)

\[ V_{DE} = \text{net vertical force leftward of section} \]
\[ V_{DE} = R_A - 100 \times (x - 40) \]
\[ V_{DE} = 4666.667 - 100(x - 40) \]

If \( V_{DE} \) will be zero at
\[ 4666.667 - 100(x - 40) = 0 \]
\[ x = 86.667 \text{ cm} \]

(c) In EB (from E to just left of B)

\[ V_{EB} = R_A - 100 \times 100 = -5333.333 \text{ kg} \]
(a) From B to C:

\[ V_{BC} = R_A - 100 \times 100 + R_B \]
\[ = (R_A + R_B) - 100 \times 100 \]
\[ = 1000 \text{ kg.} \]

`constant` \[ \uparrow \rightarrow \text{Rightward Equilibrium} \]

**Drawing B.M.O.**

(b) DE:

Net Moment: \[ M_{AD} = + R_A x \ (B.M.) \ (Linear) \]

Net Moment: \[ M_{DE} = R_A x - 100 \times (x-40) \times x - 40 \]
\[ \frac{2}{x} \]

Direction of \( M_{DE} \) (B.M. in DE) will be opposite of Direction of Net Moment.

B.M. will be \( \text{Net B = Zero in DE} \)

Because \( M_{DE} \) is \text{tue at Both} \( x = 40 \) (at B) \& \( x = 140 \) (at E)

classmate
In $EB$ -

\[ M_{EB} = R_A x - 100 \times 100 x \left( x - (40 + 50) \right) \]

\[ = R_A x - 10^4 x (x - 90) \quad \text{(Linear)} \]

$A + E \; (x = 140)$

\[ M_{EB} = 4666.67 x \times 140 - 10^4 (140 - 90) \]

\[ = 153.33 x \times 10^2 \, \text{kg cm}, \]

$A + B \; (x = 180)$

\[ M_{EB} = 4666.67 x \times 180 - 10^4 (180 - 90) \]

\[ = -60 \times 10^2 \, \text{kg cm}. \]

As sign of $M_{EB}$ is changing

So $M_{EB}$ is zero between $E + B$.

$M_{EB}$ will be zero at

\[ 4666.67 x - 10^4 (x - 90) = 0 \]

\[ 10^4 x - 4666.67 x = 10^4 x 90 \]

\[ 5333.33 x = 10^4 x 90 \]

\[ x = 168.75 \, \text{cm} \]

In $BC$ -

\[ M_{BC} = 1000 \times x \quad \text{(Linear)} \]
Q. 4.

\[ R_a = \frac{\mu_1 - \mu_2}{L} \]

\[ \mu_1 > \mu_2 \]

\[ \mu_2 - \mu_1 \frac{L}{L} \]

\[ + ve \]

\[ \mu_1 - \mu_2 \frac{L}{L} \]

\[ S.F.D. \]

\[ B.M.D. \]
Q.11.

Equation:

\[ S.F. = \frac{1}{2} \int \frac{w_0}{L} x \times x \times x = \frac{1}{4} \frac{w_0 x^2}{L} \]

(parabolic)

B.M. = \( \frac{1}{2} \int \frac{w_0 x^3}{L} \) 
\[ = \frac{w_0 x^3}{6} \]

(cubic parabola)

Q.12.

Equation:

\[ R_{AB} = \frac{1}{2} x \lambda x \times \frac{a}{2} \]

\[ R_A = \frac{w L}{3} \]

\[ P_B = \frac{w L}{6} \]

B.M. = \[ R_{AB} - \frac{w L}{L} x (L-x) x x \]
\[ - \frac{1}{2} x \times x \times (w - 2w (L-x)) \]

S.F. = \[ R_{AB} - \frac{w L}{L} x x - \frac{w (L-x)}{2} \]

S.F. will be zero at where:

\[ s.f. = 0 \]

Find \( x = ? \)

B.M. = \[ R_{AB} - \frac{w L}{L} x x x \frac{x}{2} \]
\[ - \frac{1}{2} x \times x \times (w - 2w (L-x)) \]

\( \frac{3}{x} \)

(find B.M. max. where s.f. = 0)
1. Draw the shear force and bending moment diagram and label the values of the largest positive and negative shearing force and bending moments for the beams with overhang as shown in Figure. [SSC JE - 2014]

2. Draw S.F. and B.M. diagram for beam loaded with varying load as shown in Fig. [SSC JE - 2009]

3. Draw the shear force and bending moment diagrams for the beam carrying loads as shown in Figure and locate the point of contraflexure. [SSC JE - 2005]
4. A horizontal beam ABC of total length 5m is simply supported over the span AB (length 4m) and overhang BC (length 1m) as shown in fig. The beam carries a concentrated load of 50 kN at midpoint of span AB and a uniformly distributed load of 40 kN/m over the overhang portion BC. Draw the bending moment and shear force diagrams indicating values at significant points. [SSC JE - 2004]

5. (a) Draw S.F. and B.M. diagrams for the beam as shown in Fig. [SSC JE - 2010]

6. Draw the shear force and bending moment diagrams for the beam shown in fig. [SSC JE - 2012]